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Mathematics
Higher level
Paper 3 – discrete mathematics

Tuesday 10 November 2020 (afternoon)

1 hour

Instructions to candidates

- Do not open this examination paper until instructed to do so.
- Answer all the questions.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A graphic display calculator is required for this paper.
- A clean copy of the **mathematics HL and further mathematics HL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[50 marks]**.

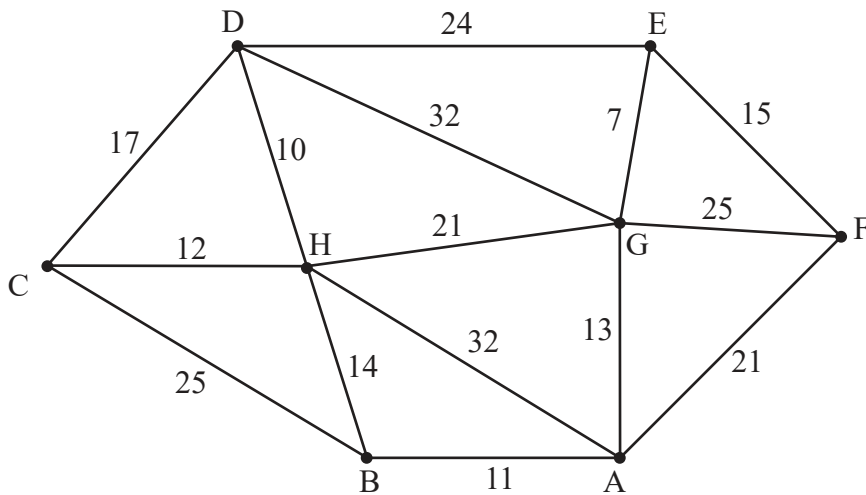
Please start each question on a new page. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. [Maximum mark: 8]

Solve the recurrence relation $u_{n+2} = 8u_{n+1} - 16u_n$, given that $u_1 = 4$ and $u_2 = 48$.

2. [Maximum mark: 12]

Christine and her friends live in Winnipeg, Canada. The weighted graph shows the location of their houses and the time, in minutes, to travel between each house.



Christine's house is located at vertex C.

(a) (i) Use Dijkstra's algorithm to find the shortest time to travel from C to F, clearly showing how the algorithm has been applied.

(ii) Hence write down the shortest path from C to F. [7]

A new road is constructed that allows Christine to travel from H to A in t minutes. If Christine starts from home and uses this new road her minimum travel time to A is reduced, but her minimum travel time to F remains the same.

(b) Find the possible values of t . [5]

3. [Maximum mark: 13]

- (a) (i) Write down the remainder when 14^{2022} is divided by 7.
- (a) (ii) Use Fermat's little theorem to find the remainder when 14^{2022} is divided by 17. [5]
- (b) (i) Prove that a number in base 13 is divisible by 6 if, and only if, the sum of its digits is divisible by 6.
- (b) (ii) The base 13 number $1y93y25$ is divisible by 6. Find the possible values of the digit y . [8]

4. [Maximum mark: 6]

Solve the system of linear congruences

$$x \equiv 7 \pmod{29} \text{ and } 3x \equiv 1 \pmod{31}.$$

5. [Maximum mark: 11]

G is a simple, connected, planar graph with 9 vertices and e edges.

- (a) Find the maximum possible value of e . [2]

The complement of G has e' edges.

- (b) Find an expression for e' in terms of e . [2]
- (c) Given that the complement of G is also planar and connected, find the possible values of e . [2]

H is a simple graph with v vertices and e edges.

- (d) Given that both H and its complement are planar and connected, find the maximum possible value of v . [5]